

$\Omega_b \rightarrow \Omega_c, \Omega_c^*$ transitions:
Model-independent bounds on invariant form factors.

J.G.Körner¹, K.Melnikov^{2*}, O.Yakovlev^{3**}

Johannes Gutenberg-Universität
Institut für Physik (THEP), Staudingerweg 7
D-55099 Mainz, Germany

Abstract

In this note we report some model-independent bounds involving transition form factors for $\Omega_b \rightarrow \Omega_c$ and $\Omega_b \rightarrow \Omega_c^*$ and the nonperturbative matrix elements of the Ω_b system. They are derived by using operator product expansion (OPE) in Heavy Quark Effective Theory .

[5cm] ¹ Supported in part by the BMFT, Germany, under contract 06MZ730

² Supported by the Graduiertenkolleg Teilchenphysik, Universität Mainz

³ Supported by Deutsche Forschungsgemeinschaft

^{*)} on leave of absence from Novosibirsk State University, Novosibirsk, Russia

^{**)} on leave of absence from Budker Institute for Nuclear Physics, Novosibirsk, Russia

1. Recently it has been shown how to obtain model-independent bounds on the formfactors that describe semileptonic decays of heavy hadrons. In what follows, we extend previous results on mesonic transitions $B \rightarrow D, D^*$ [1-3] and the baryonic transitions $\Lambda_b \rightarrow \Lambda_c$ [4] to the baryonic transitions $\Omega_b \rightarrow \Omega_c, \Omega_c^*$.

2. As usual, we start from the time-ordered product of two appropriate currents (vector or axial) inducing $b \rightarrow c$ transitions.

$$T_{\mu\nu} = -i \int d^4x e^{-iq \cdot x} J^+_\mu(x) J_\nu(0) \quad (1)$$

The best way to obtain the appropriate operator product expansion is to integrate out the intermediate quark field which reduces Eq.(1) to:

$$T_{\mu\nu} = -i \int d^4x e^{-iq \cdot x} \langle x | \bar{Q} \Gamma \frac{1}{i\hat{D} - m_c + i\epsilon} Q | 0 \rangle \quad (2)$$

Here $\langle x |$ and $| 0 \rangle$ stand for states with definite space coordinates. The usual HQET fermion field redefinition $Q = e^{im_b v \cdot X} h$ results in the simple replacement of $iD \rightarrow m_b v + iD$ in Eq.(2). (Here X stands for the four-space coordinate operator). Then rewriting $e^{-iq \cdot x} \langle x |$ as $\langle x | e^{-iq \cdot X}$ we finally get the well-known formulae which is at the basis of the operator product expansion method in HQET [11]:

$$T_{\mu\nu} = -i \int d^4x \langle x | \bar{Q} \Gamma \frac{1}{m_b \hat{v} - \hat{q} + i\hat{D} - m_c + i\epsilon} Q | 0 \rangle \quad (3)$$

As is seen from the redefinition of the quark field, the covariant derivative in Eq.(3) corresponds to the "residual" momenta of the heavy quark inside the heavy hadron. The natural assumption for this quantity is to be of $O(\Lambda_{QCD})$. Assuming also $q \approx m_b$ we recognize the possibility to expand Eq.(3) in powers of $(\frac{iD}{m_b})$. In what follows we will expand Eq.(3) up to second order in $(\frac{iD}{m_b})$ and consider matrix element of the time-ordered product between Ω_b states.

3. Expanding Eq.(3) up to second order in $(\frac{iD}{m_b})$ we obtain terms with two, one or no derivatives. Let us generically write the corresponding matrix elements as

$$\langle \Omega_c | \bar{h} \Gamma_1 h | \Omega_c \rangle \quad (4)$$

$$\langle \Omega_c | \bar{h} \Gamma_1 i D_\alpha h | \Omega_c \rangle \quad (5)$$

$$\langle \Omega_c | \bar{h} \Gamma_1 i D_\alpha i D_\beta h | \Omega_c \rangle \quad (6)$$

where Γ_1 stands for an arbitrary 4×4 matrix in spinor space.

Now we are in the position to discuss how to compute these matrix elements. To begin with, let us discuss the matrix element (Eq.(6)) with two derivatives. As we are interested in the power corrections of order $(\frac{\Lambda_{QCD}}{m_b})^2$ we need this matrix

element only up to the zeroth order in this parameter. It is then straightforward to eliminate residual h fields in favour of the HQET fields h_v and to use the spin-flavour symmetry together with the lowest order HQET equations of motion to parametrize this matrix element. We get:

$$\begin{aligned} \frac{\langle \Omega_b | \bar{h} \Gamma_1 i D_\alpha i D_\beta h | \Omega_b \rangle}{m_b^2} &= u_1 \cdot (g_{\alpha\beta} - v_\alpha v_\beta) \bar{R}^a \Gamma_1 R_a \\ + u_2 \cdot (\bar{R}_\alpha \Gamma_1 R_\beta + \bar{R}_\beta \Gamma_1 R_\alpha) &+ u_3 \cdot (\bar{R}_\alpha \Gamma_1 R_\beta - \bar{R}_\beta \Gamma_1 R_\alpha) \end{aligned} \quad (7)$$

Here R^a stands for the Ω_b state in HQET (see Refs. [5-9]). The constants u_1, u_2, u_3 , appearing in Eq.(7) are of order $(\frac{\Lambda_{QCD}}{m_b})^2$. Note that in contrast to the previously treated meson and baryon transitions, here we need one more invariant matrix element to parametrize Eq.(7). The reason is that the total angular momenta of the light diquark system in the hadron is equal to one vs. $J_{light} = \frac{1}{2}$ and $J_{light} = 0$ in the previously studied cases. Taking for example Γ_1 to be the unity matrix and contracting Eq.(7) with the metric tensor, we obtain:

$$\frac{\langle \Omega_b | \bar{h} (iD)^2 h | \Omega_b \rangle}{m_b^2} = -(3u_1 + 2u_2) \quad (8)$$

Using the normalization $\bar{R}^a R_a = -1$ the quantity on the left hand side of Eq.(8) represents the heavy quark kinetic energy inside the Ω_b baryon. A similar quantity was introduced earlier for mesons in [1-3] and estimated recently in [11]. Here we use similar notation and define:

$$\frac{\langle \Omega_b | \bar{h} (iD)^2 h | \Omega_b \rangle}{m_b^2} = -(3u_1 + 2u_2) = \frac{-\mu_\pi^2}{m_b^2} \quad (9)$$

Analogously, setting Γ_1 equal to $2i\sigma_{\alpha\beta}$ we arrive at:

$$\frac{-\langle \Omega_b | \bar{h} \sigma_{\alpha\beta} g_{QCD} G^{\alpha\beta} h | \Omega_b \rangle}{m_b^2} = 2u_3 R^\beta \sigma_{\alpha\beta} R^\alpha = 8u_3 = 2\frac{\mu_G^2}{m^2} \quad (10)$$

Here we have introduced an additional quantity μ_G^2 , which can be related to the mass difference of the Ω_b and Ω_b^* baryons:

$$m_{\Omega_b^*} - m_{\Omega_b} = \frac{3\mu_G^2}{8m_b} \quad (11)$$

Next let us discuss the calculation of the matrix element Eq. (5) containing one derivative. This matrix element is only of order $\frac{\Lambda_{QCD}}{m_b}$, hence we have to calculate it in the next to leading order in this parameter. First we expand the QCD fields up to the first order in $\frac{\Lambda_{QCD}}{m_b}$:

$$h(x) = (1 + \frac{i\hat{D}}{2m})h_v(x) \quad (12)$$

We then arrive at the following expression:

$$\begin{aligned} \langle \Omega_b | \bar{h} \Gamma_1 i D_\alpha h | \Omega_b \rangle &= \langle \Omega_b | \bar{h}_v \Gamma_1 i D_\alpha h_v | \Omega_b \rangle + \\ &\quad \frac{1}{2m_b} \langle \Omega_b | \bar{h}_v (-i \hat{D} \Gamma_1 i D_\alpha + \Gamma_1 i D_\alpha i \hat{D}) h_v | \Omega_b \rangle \end{aligned} \quad (13)$$

However, it can be shown that the first term on the r.h.s. of Eq.(13) is of order $(\frac{\Lambda_{QCD}}{m_b})^2$. The proof is based on the equation of motion of the heavy quark and the observation that the first term on the r.h.s. is proportional to the velocity of the heavy quark up to the required accuracy. In this way we finally obtain:

$$\begin{aligned} \langle \Omega_b | \bar{h} \Gamma_1 i D_\alpha h | \Omega_b \rangle &= \frac{(-v_\alpha \langle \Omega_b | \bar{h}_v \Gamma_1 (i \hat{D}_\perp)^2 h_v | \Omega_b \rangle)}{2m_b} \\ &+ \frac{\langle \Omega_b | \bar{h}_v (-i \hat{D} \Gamma_1 i D_\alpha + \Gamma_1 i D_\alpha i \hat{D}) h_v | \Omega_b \rangle}{2m_b} \end{aligned} \quad (14)$$

This last expression has to be computed up to the leading order in $\frac{\Lambda_{QCD}}{m_b}$, consequently we can use the Eq.(7) to rewrite the result in terms of the parameters u_1 , u_2 and u_3 .

Let us finally consider the matrix element Eq.(4) with no derivatives $\langle \Omega_b | \bar{h} \Gamma_1 h | \Omega_b \rangle$. In order to obtain this matrix element up to the required order one has to expand $\bar{h} \Gamma_1 h$ up to the second order in the inverse powers of the quark mass and one also has to take into account the difference between the QCD and HQET wave functions of the final and initial states. In this way we obtain the following parametrization for the full basis of $4 * 4$ matrices:

$$\langle \Omega_b | \bar{h} h | \Omega_b \rangle = 1 - \frac{\mu_\pi^2}{2m_b^2} + \frac{\mu_G^2}{2m_b^2} \quad (15)$$

$$\langle \Omega_b | \bar{h} \gamma_\mu h | \Omega_b \rangle = v_\mu \quad (16)$$

$$\langle \Omega_b | \bar{h} \gamma_\mu \gamma_5 h | \Omega_b \rangle = \frac{-s_\mu}{3} (1 + \frac{\mu_s^2}{m_b^2}) \quad (17)$$

$$\langle \Omega_b | \bar{h} \sigma_{\mu\nu} h | \Omega_b \rangle = \frac{i \epsilon_{\mu\nu\alpha\beta} v_\alpha s_\beta}{3} (1 + \frac{\mu_s^2}{m_b^2} + \frac{u_1 + 14u_2 - 4u_3}{2}) \quad (18)$$

$$\langle \Omega_b | \bar{h} \gamma_5 h | \Omega_b \rangle = 0 \quad (19)$$

4. Using the parametrization of the matrix elements discussed in the previous section it is straightforward to compute $\langle \Omega_b | T_{\mu\nu} | \Omega_b \rangle$ up to the necessary order after

expanding equation (3) in terms of $\frac{iD}{m}$. The result for the invariant form factors (which are defined in full analogy with refs. [2,3,4]) are too lengthy to be presented here. What is really of interest is the zero-recoil projection of these quantities onto the helicity structure functions. We thus compute $n_\mu^{*(\lambda)} n_\nu^\lambda \int d(q \cdot v) Im \langle \Omega_b | T_{\mu\nu} | \Omega_b \rangle$ at the point of zero recoil. Here $n^{(\lambda)}$ stands for the set of polarization vectors of the outgoing particle with helicity λ (say, W -boson for the weak-current case). This quantity is positive definite since it is in one-to-one correspondence with the particle decay width into the diagonal helicity states of the off-shell W 's.

5. On the other hand, we can express $T_{\mu\nu}$ in terms of the phenomenological form factors which describe the $\Omega_b \rightarrow \Omega_c$, $\Omega_b \rightarrow \Omega_c^*$ and $\Omega_b \rightarrow$ excited states transitions. Again, projecting "hadronic" tensor $T_{\mu\nu}$ onto helicity states, taking the imaginary part and integrating over $q \cdot v$ we arrive at the positive definite quantities W_L , $W_{T_{L,R}}$ and W_0 . (for further details see ref. [4]). The exact expressions of Ω_c and Ω_c^* contributions to this quantities for the case of axial and vector current are presented in the Appendix.

6. As a next point let us discuss the sum rules for the form factors.

Taking linear combination $\frac{1}{2}(W_{T_L} + W_{T_R})$ for the hadron-side and parton-side contributions and neglecting the contributions from the excited states, we finally get the inequality:

$$|f_1^A|^2 + \frac{2}{3}|G_1^A|^2 \leq 1 - \frac{\mu_\pi^2}{m_b^2} \left(\frac{x^2}{4} + \frac{x}{6} + \frac{1}{4} \right) - \frac{\mu_G^2}{m_b^2} \left(\frac{x^2}{12} - \frac{x}{6} - \frac{1}{4} \right) \quad (20)$$

Here x stands for the ratio $\frac{m_b}{m_c}$.

On the other hand, taking $n \cdot s = -1$ and calculating W_{T_L} we can "switch off" the Ω_b state contribution on the hadron side, thus obtaining a sum rule for the G_1^A formfactor only:

$$\frac{1}{2}|G_1^A|^2 \leq \frac{2}{3} - \frac{\mu_s^2}{3m_b^2} + \frac{\mu_G^2}{m_b^2} \left(\frac{1}{3} + \frac{x}{6} \right) - \frac{\mu_\pi^2}{m_b^2} \left(\frac{x^2}{6} + \frac{x}{6} + \frac{1}{4} \right) + u_1 \left(\frac{x}{6} - \frac{1}{12} \right) - u_2 \left(x + \frac{14}{12} \right) \quad (21)$$

Note that u_1 and u_2 enter this sum rule as independent quantities i.e. they do not enter in the combination of kinetic energy (7).

In full analogy we can obtain a bound for the vector form factors. It is worth noting, that at zero recoil, the Ω_c^* state does not contribute to the vector-current induced transition (see Appendix). Thus we get:

$$|\Sigma f_i^V|^2 \leq 1 - \frac{(\mu_\pi^2 - \mu_G^2)}{4m_b^2} (x - 1)^2 \quad (22)$$

As Ω_c^* does not contribute to the vector current induced transitions, we can also think of "switching off" the contribution from the Ω_c state. We can do that by taking the first moment while integrating over $q \cdot v$ (similar to the Voloshin sum

rule , but at zero recoil). The leading contributions to the " hadronic " side of the sum rules will be zero in this way , while we get something none zero on the partonic side. Again neglecting the contributions from the excited states we get:

$$\mu_G^2 \leq \mu_\pi^2 \quad (23)$$

We mention here that a similar inequality was obtained in the Ref.[11] for mesonic states using different techniques.

7. To summarize, we have estimated the size of $(\frac{\Lambda_{QCD}}{m_b})^2$ corrections to Ω_b to Ω_c transitions at zero recoil point using the operator product expansion in HQET.

8. **Acknowledgments.** The authors are grateful to Dan Pirjol for useful conversations.

Appendix.

The contribution of Ω_c , Ω_c^* to the projection of the hadronic tensor to the helicity structure functions:

$$\Omega_b \rightarrow \Omega_c$$

1.Axial current:

$$\langle \Omega_c(v', s') | \bar{c}\gamma_\mu\gamma_5 b | \Omega_b(v, s) \rangle = \bar{u}_c(v', s')[f_1^A\gamma_\mu + f_2^A v_\mu + f_3^A v'_\mu]\gamma_5 u_b(v, s);$$

$$W_L = \frac{w+1}{2q^2}((m_{\Omega_b} - m_{\Omega_c})f_1^A - m_{\Omega_c}(w-1)f_2^A - m_{\Omega_b}(w-1)f_3^A)^2;$$

$$W_{T_{L,R}} = |f_1^A|^2 \frac{w+1}{2}(1 \pm \vec{s}\vec{n});$$

$$W_0 = \frac{w-1}{2q^2}((m_{\Omega_b} + m_{\Omega_c})f_1^A - (m_{\Omega_b} - m_{\Omega_c}w)f_2^A - (m_{\Omega_b}w - m_{\Omega_c})f_3^A)^2;$$

2.Vector current:

$$\langle \Omega_c(v', s') | \bar{c}\gamma_\mu b | \Omega_b(v, s) \rangle = \bar{u}_c(v', s')[f_1^V\gamma_\mu + f_2^V v_\mu + f_3^V v'_\mu]u_b(v, s);$$

$$W_L = \frac{w-1}{2q^2}((m_{\Omega_b} + m_{\Omega_c})f_1^V + m_{\Omega_c}(w+1)f_2^V + m_{\Omega_b}(w+1)f_3^V)^2;$$

$$W_{T_{L,R}} = |f_1^V|^2 \frac{w-1}{2}(1 \pm \vec{s}\vec{n});$$

$$W_0 = \frac{w+1}{2q^2} ((m_{\Omega_b} - m_{\Omega_c}) f_1^V + (m_{\Omega_b} - m_{\Omega_c} w) f_2^V + (m_{\Omega_b} w - m_{\Omega_c}) f_3^V)^2;$$

$$\Omega_b \rightarrow \Omega_c^*$$

1.Axial current:

$$\begin{aligned} <\Omega_c^* | \bar{c}\gamma_\mu\gamma_5 b | \Omega_b> &= \bar{u}_c^\nu(v_2)[G_1^A g_{\nu,\mu}\gamma_5 + G_2^A v_{1\nu}\gamma_\mu\gamma_5 \\ &+ G_3^A v_{1\nu}v_{1\mu}\gamma_5 + G_4^A v_{1\nu}v_{2\mu}\gamma_5]\gamma_5 u_b(v_1); \end{aligned}$$

$$\begin{aligned} W_L &= \frac{w+1}{3q^2} ((m_{\Omega_b} w - m_{\Omega_c^*}) G_1^A + (m_{\Omega_c^*} + m_{\Omega_b})(w-1) G_2^A \\ &+ m_{\Omega_c^*}(w^2-1) G_3^A + m_{\Omega_b}(w^2-1) G_4^A)^2; \end{aligned}$$

$$\begin{aligned} W_0 &= \frac{(w-1)(w+1)^2}{3q^2} (m_{\Omega_b} G_1^A + (m_{\Omega_b} - m_{\Omega_c}) G_2^A \\ &+ (m_{\Omega_b} - m_{\Omega_c^*} w) G_3^A + (m_{\Omega_b} w - m_{\Omega_c^*}) G_4^A)^2; \end{aligned}$$

$$\begin{aligned} W_{T_{L,R}} &= \frac{w+1}{4} (\frac{1}{3}(G_1^A - 2(w-1)G_2^A)^2 + (G_1^A)^2 \\ &\pm \vec{s}\vec{n}(\frac{1}{3}(G_1^A - 2(w-1)G_2^A)^2 - (G_1^A)^2); \end{aligned}$$

2.Vector current:

$$\begin{aligned} <\Omega_c^* | \bar{c}\gamma_\mu b | \Omega_b> &= \bar{u}_c^\nu(v_2)[G_1^V g_{\nu,\mu} + G_2^V v_{1\nu}\gamma_\mu \\ &+ G_3^V v_{1\nu}v_{1\mu} + G_4^V v_{1\nu}v_{2\mu}]\gamma_5 u_b(v_1); \end{aligned}$$

$$\begin{aligned} W_L &= \frac{w-1}{3q^2} ((m_{\Omega_b} w - m_{\Omega_c^*}) G_1^V - (m_{\Omega_b} - m_{\Omega_c^*})(w+1) G_2^V \\ &+ m_{\Omega_c^*}(w^2-1) G_3^V + m_{\Omega_b}(w^2-1) G_4^V)^2; \end{aligned}$$

$$\begin{aligned} W_0 &= \frac{(w+1)(w-1)^2}{3q^2} (m_{\Omega_b} G_1^V - (m_{\Omega_b} + m_{\Omega_c^*}) G_2^V \\ &+ (m_{\Omega_b} - m_{\Omega_c^*} w) G_3^V + (m_{\Omega_b} w - m_{\Omega_c^*}) G_4^V)^2; \end{aligned}$$

$$\begin{aligned} W_{T_{L,R}} &= \frac{w-1}{4} (\frac{1}{3}(G_1^V - 2(w+1)G_2^V)^2 + (G_1^V)^2 \\ &\pm \vec{s}\vec{n}(\frac{1}{3}(G_1^V - 2(w+1)G_2^V)^2 - (G_1^V)^2); \end{aligned}$$

References

1. J.Chay, H.Georgi, B.Grinstein, Phys. Lett. B247 (1990) 399.
2. I.Bigi, M.Shifman, N.Uraltsev, A.Vainstein, Phys. Rev. Lett. 71,(1993) 496 .
“B.Blok, L.Koynakh, M.Shifman, A.Vainstein, Phys. Rev. D49 (1994) 3356.
3. A.Manohar, M.Wise , Phys. Rev. D49 (1994) 1310.
4. J.G. Körner, D.Pirjol, preprint MZ-TH/94-17, May 94.
5. H.Georgi, Nucl. Phys. B348 (1991) 293.
6. T.Mannel, W.Roberts, Z.Ryzak, Nucl. Phys. B355, (1991) 38.
7. C.G Boyd, D.E.Brahm Phys. Lett. B257 (1991) 393.
8. A.Falk, Nucl. Phys. B378 (1992) 80.
9. J.G. Körner, D.Pirjol, M. Krämer, Preprint, DESY 94-095, MZ-THEP-94-08.
10. I. Bigi, A.Grozin, M. Shifman, N.Uraltsev, A.Vainstein,
Preprint TPI-MINN-94/25-T.
11. M.Shifman, N.Uraltsev, A.Vainstein TPI-MINN/13-T.